

A note about a conjecture

MARIUS MIHAI BIROU
(CLUJ-NAPOCA)

ABSTRACT. The sum of the squared Bernstein polynomials is a convex function. In this note we show that this property doesn't hold for q -Bernstein polynomials, for some values of the parameter q .

KEY WORDS: q -Bernstein basis, convex function

MSC 2010: 41A36, 26A51

Let $n \geq 1$ and

$$S_n(x) = \sum_{k=0}^n p_{n,k}^2(x), \quad x \in [0, 1],$$

where

$$p_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}, \quad k = 0, \dots, n,$$

form the Bernstein basis.

In [3] the authors stated the following Conjecture:

Conjecture. The function S_n is a convex function in the interval $[0, 1]$.

The Conjecture was confirmed in [4] using properties of the Legendre polynomials.

[◇]Marius Mihai Birou, Technical University of Cluj-Napoca, Memorandumului str. 28-30, Cluj-Napoca, Romania,
email: Marius.Birou@math.utcluj.ro

In [2] the authors also confirmed the Conjecture. They give an explicit representation for the function S_n . Moreover, they prove that the function S_n is convex for other choices instead of the Bernstein basis: the Mirakjan-Favard-Szasz basis, the Meyer-Konig and Zeller basis. For the King basis they have shown that the sums S_n are not convex for all $n \geq 1$.

In the last period the q -operators were intensively studied (see [1]).

Let $q \in (0, 1)$ and $n \in \mathbb{N}$. We remind some notations from q -calculus:

- q -integer

$$[n]_q = \frac{1 - q^n}{1 - q},$$

- q -factorial

$$[n]_q! = [n]_q [n-1]_q \dots [1]_q, \quad n = 1, 2, \dots, \quad [0]_q! = 1,$$

- q -binomial coefficients

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[k]_q! [n-k]_q!}, \quad 0 \leq k \leq n.$$

Let $n \geq 1$ and

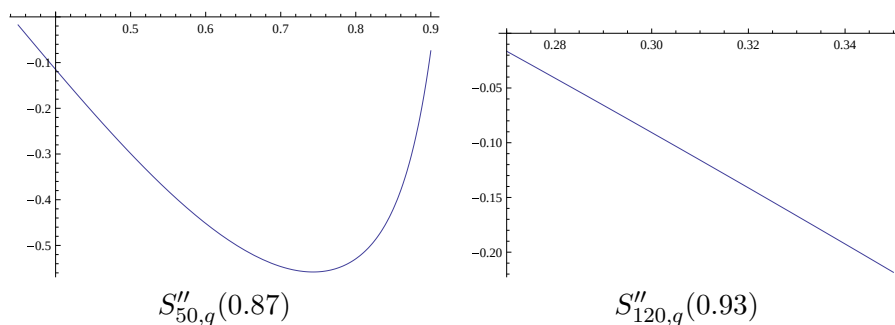
$$S_{n,q}(x) = \sum_{k=0}^n p_{n,k}^2(q; x), \quad x \in [0, 1], \quad q \in (0, 1),$$

where

$$p_{n,k}(q; x) = \begin{bmatrix} n \\ k \end{bmatrix}_q x^k \prod_{s=0}^{n-k-1} (1 - q^s x), \quad k = 0, \dots, n,$$

form the q -Bernstein basis (see [5]). For $q = 1$ we get the case of the classical Bernstein basis.

The Conjecture is not true for the q -Bernstein basis for some $q \in (0, 1)$. We have that for $q \in [0.35, 0.9]$, $n = 50$ and $q \in [0.27, 0.35]$, $n = 120$ the second derivative of the functions $S_{n,q}$ is negative in the points 0.87 and respectively 0.93.

Figure 1: $S''_{n,q}(x)$

References

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